TEST BOARD LCR EVALUATION
(FOR DC-DC CONVERTER)

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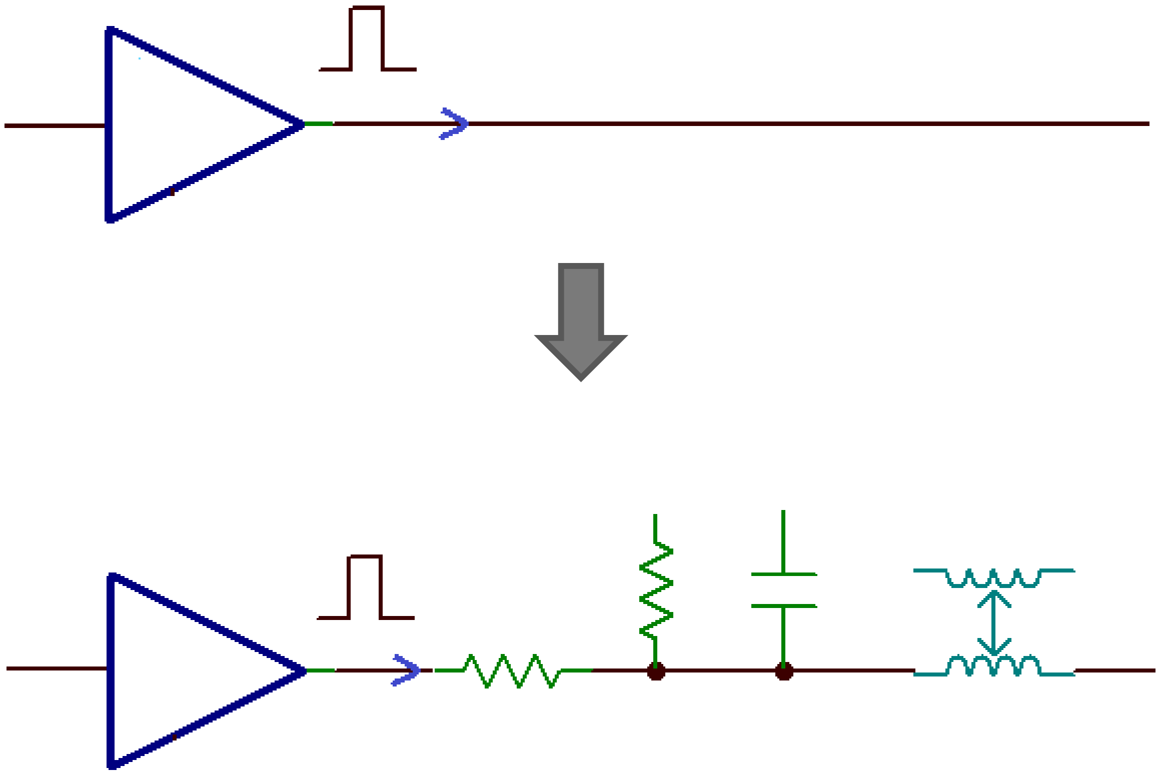
TOPICS

- **Parasitic parameters of PCB trace**
- **LCR analysis**
- **Transmission line**

FEATURE OF TEST BOARD (DC-DC)

- Long traces (test head dimension and cable)
- Wide traces (high current)
- Additional components (relays, buffers, servo-loop etc.)
- Multi-traces (multi-resources per pin)
- Multi-sites (parallel test)
- Multi-layers (noise isolation)

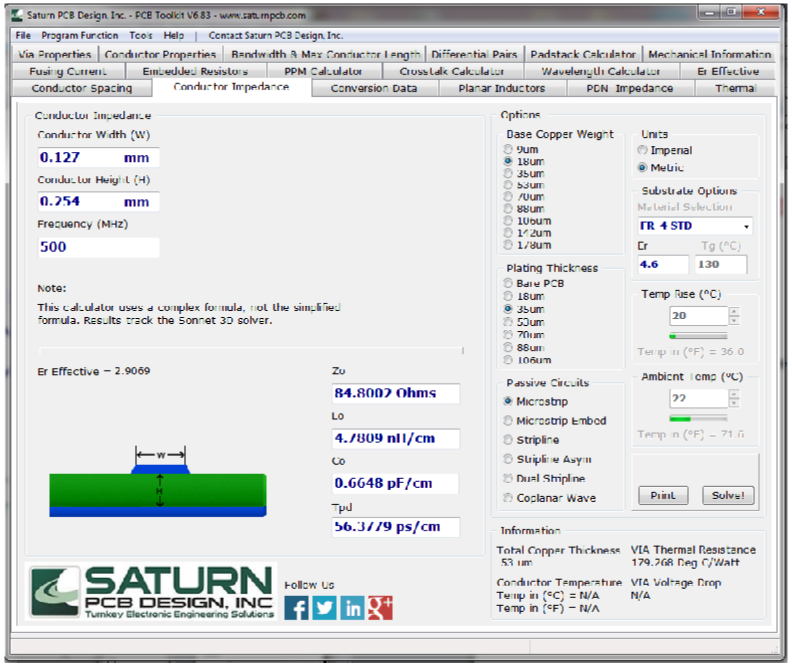
PARASITIC MODEL



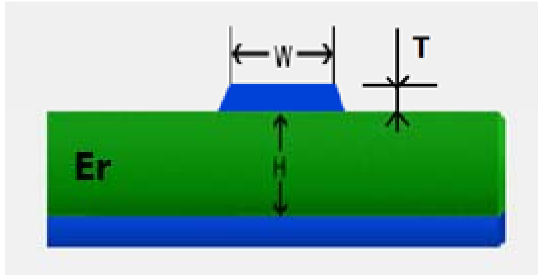
MODEL OF PCB TRACE

W: Width of trace
T: Thickness of trace
H: Distance of trace and copper plate
Er: Dielectric constant

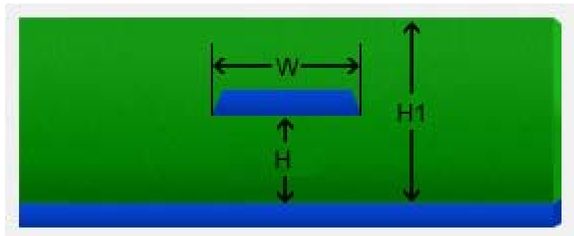
Tool: Saturn PCB Toolkit



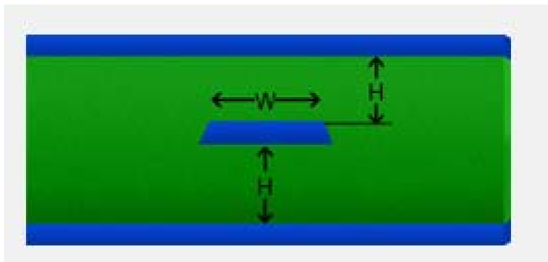
Micro-Strip



Micro-Strip Embed

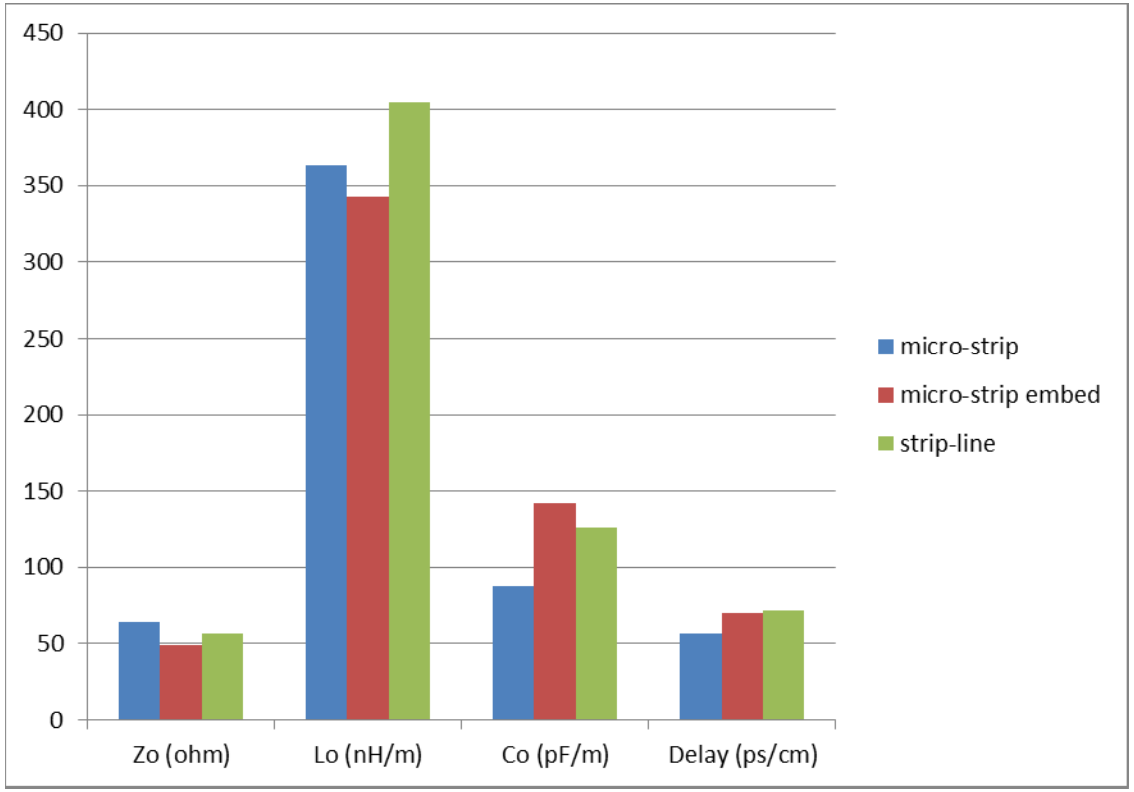


Strip-Line



COMPARISON

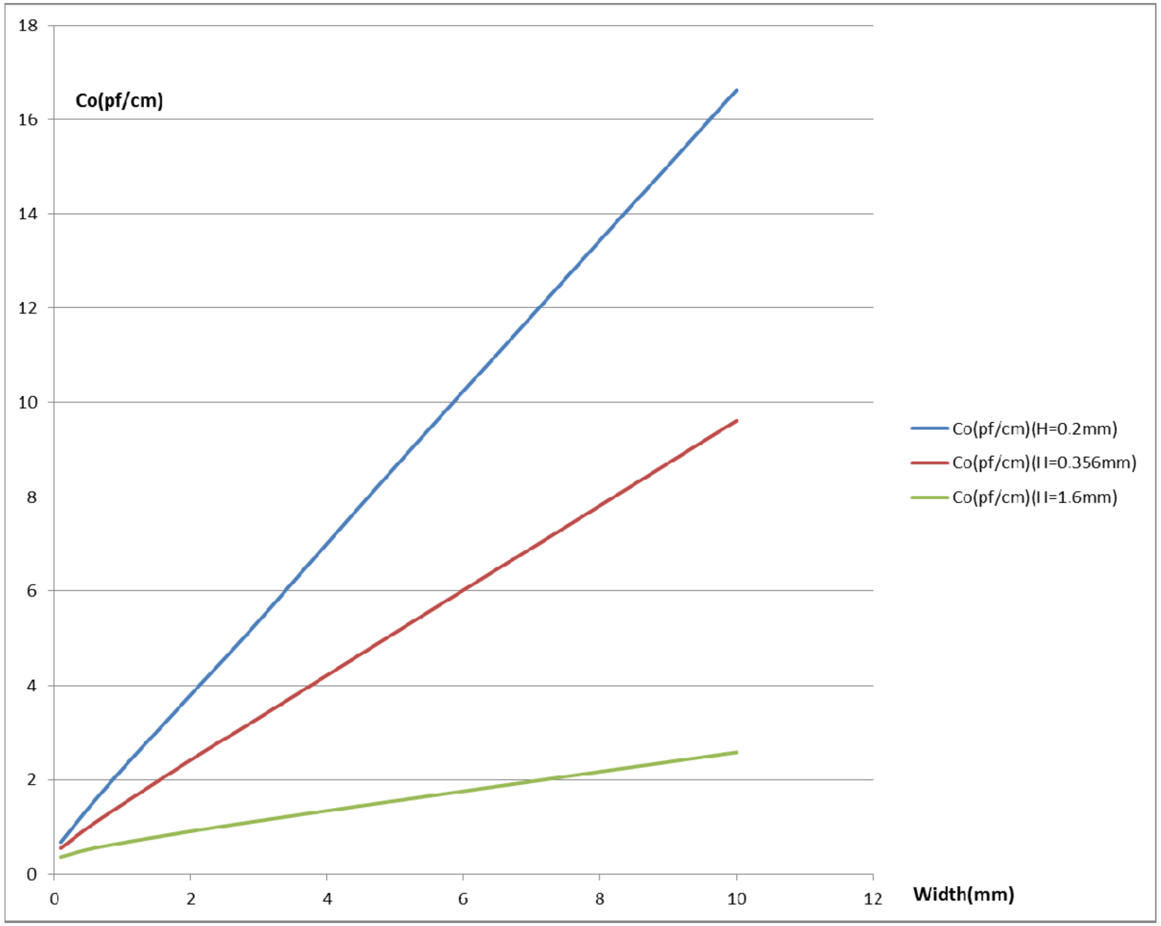
T=35um (1OZ)
ER=4.6 (FR-4)
W=0.127mm
H=0.127mm



TRACE CAPACITANCE

T=35um (1OZ)
ER=4.6 (FR-4)
Ea=8.86*10^-12F/m

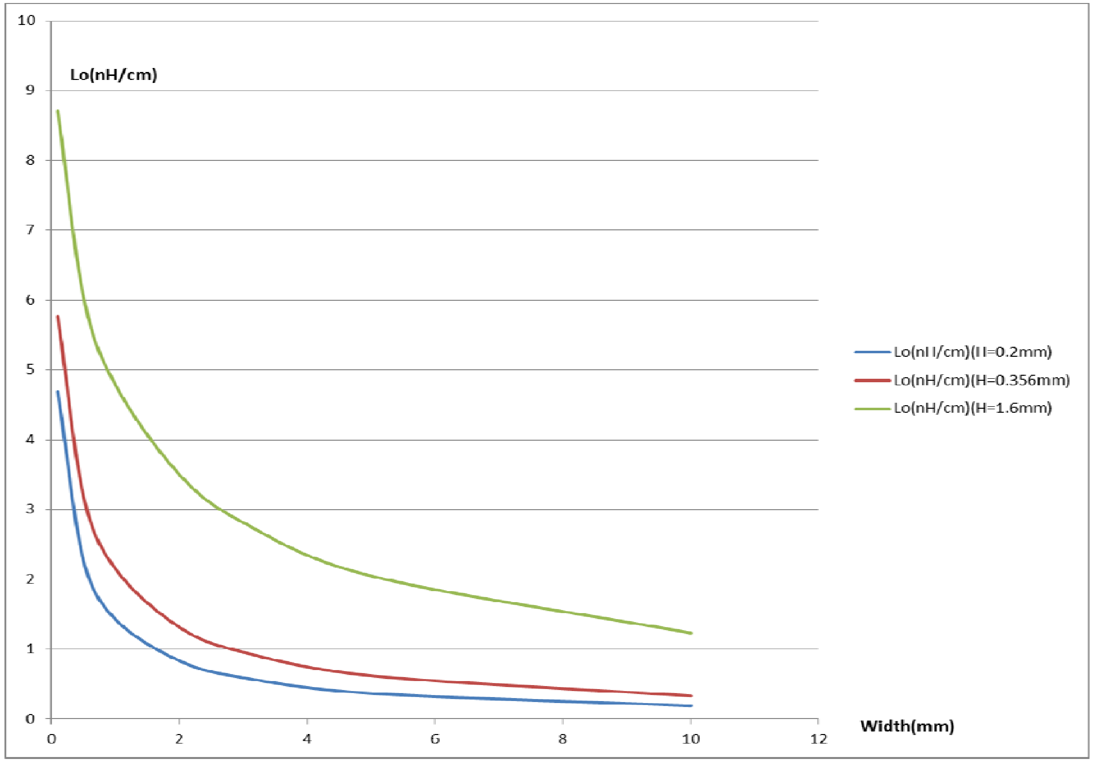
C=W*l*Ea*Er/H (F)



TRACE INDUCTANCE

T=35 um (1OZ)
ER=4.6 (FR-4)

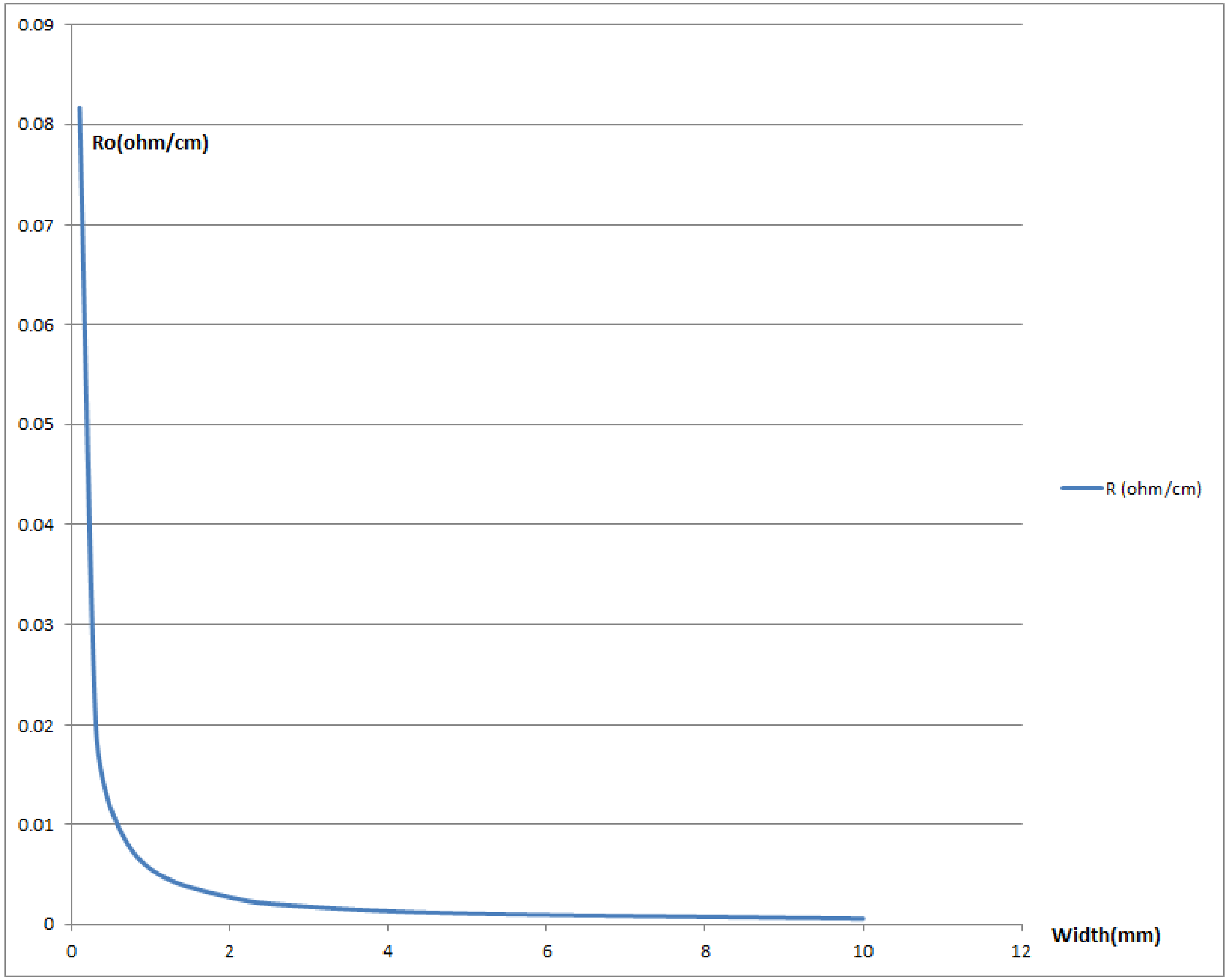
L=200*l*(ln(2l/(W+T))+0.5+0.2235(W+T)/l) (nH)



TRACE RESISTANCE

T=35um (1OZ)
ER=4.6 (FR-4)
ρCu=1.75*1e-8 Ω.m

R=ρL/S (Ω)

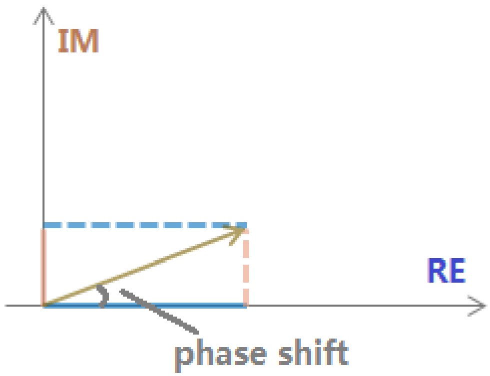


SINE WAVE RESPONSE

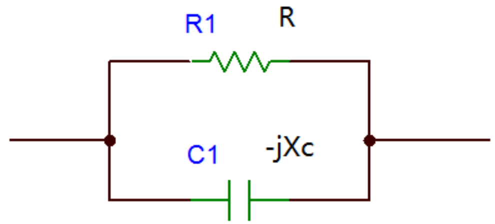
XR=R

XC=1/(2 π f C)

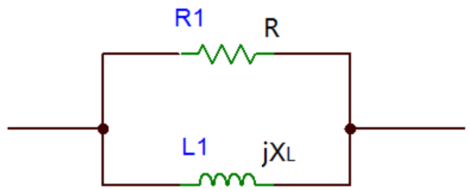
XL=2 π f L



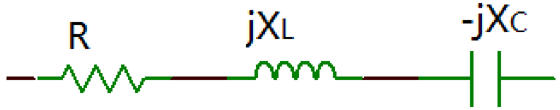
$$Z = \frac{-jXR}{(R - jX)} = (RX^2 - jXR^2)/(R^2 + X^2)$$



$$Z = \frac{jXR}{(R + jX)} = (RX^2 + jXR^2)/(R^2 + X^2)$$



$$Z = R + jX_L - jX_c$$



LCR ZERO-INPUT RESPONSE

$R^2C - 4L > 0$:

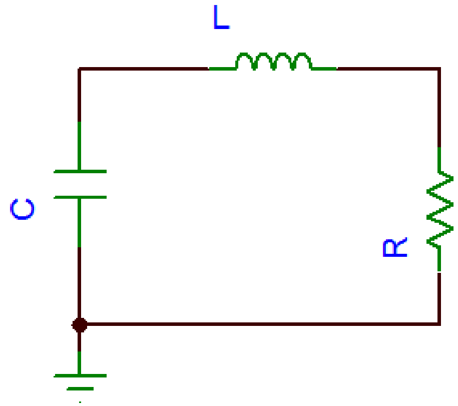
$$u_c = u_{c0}\left[\left(\frac{Li_0}{CkU_{c0}} + \frac{R}{2k} + \frac{1}{2}\right)e^{\frac{k-R}{2L}t} - \left(\frac{Li_0}{CkU_{c0}} + \frac{R}{2k} - \frac{1}{2}\right)e^{-\frac{k+R}{2L}t}\right]$$

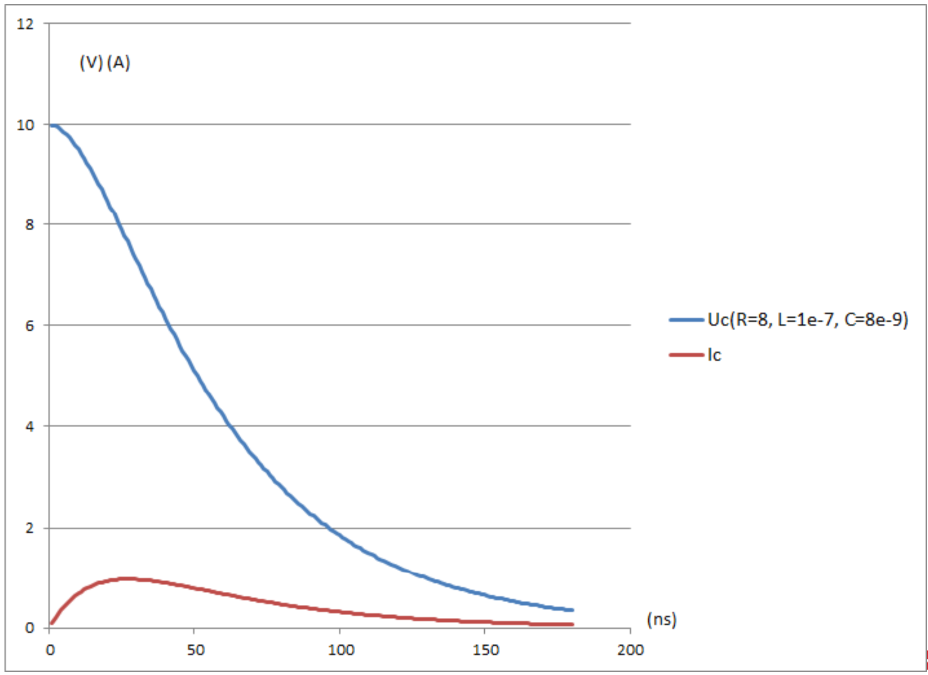
$$= u_{c0}\left[\left(\frac{Li_0}{CkU_{c0}} + \frac{R}{2k} + \frac{1}{2}\right)e^{\frac{-2}{CR+Ck}t} - \left(\frac{Li_0}{CkU_{c0}} + \frac{R}{2k} - \frac{1}{2}\right)e^{\frac{-2}{CR-Ck}t}\right]$$

$$i_c = i_0\left[\left(\frac{-R}{2k} + \frac{1}{2} - \frac{U_0}{ki_0}\right)e^{\frac{k-R}{2L}t} - \left(\frac{-R}{2k} - \frac{1}{2} - \frac{U_0}{ki_0}\right)e^{-\frac{k+R}{2L}t}\right]$$

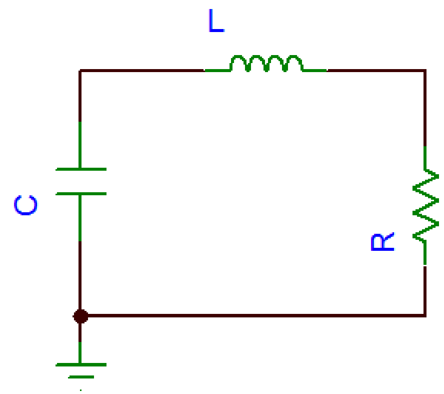
$$\beta = \sqrt{\frac{R^2C - 4L}{4CL^2}} = \frac{k}{2L} \quad \left(k = \sqrt{R^2 - 4L/C}\right)$$

$$\tau 1 = \frac{CR+Ck}{2} \quad \left(\frac{CR}{2} < \tau < CR\right)$$





LCR ZERO-INPUT RESPONSE



$R^2C - 4L < 0$:

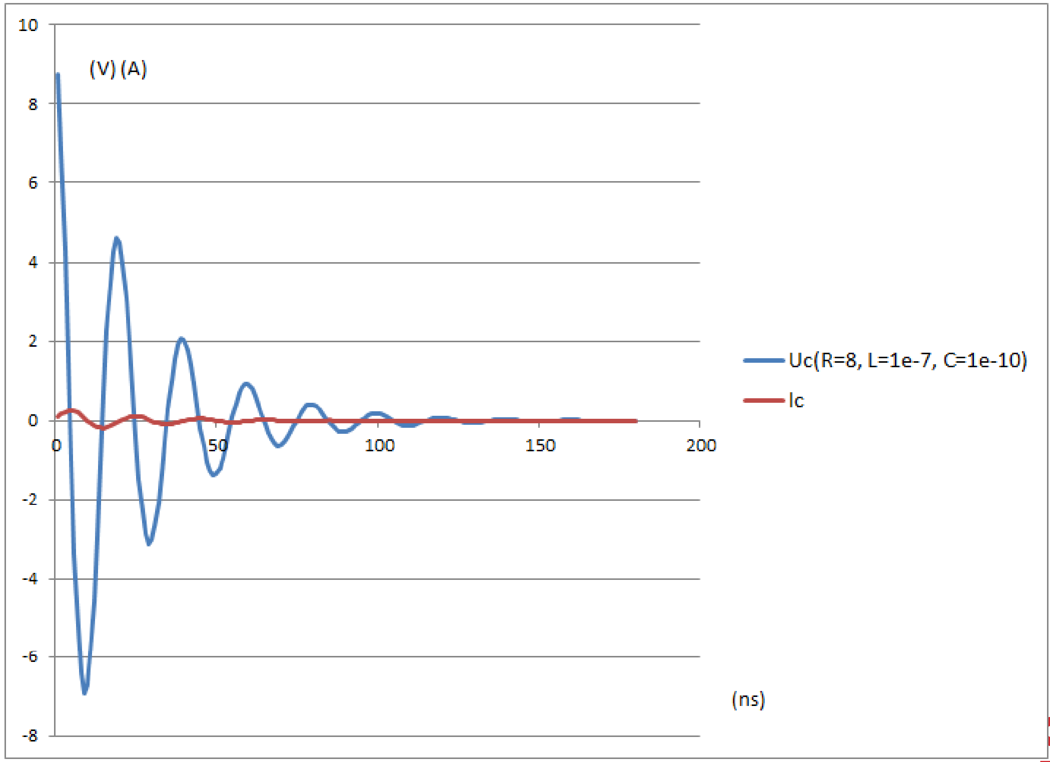
$$u_c = u_{c0}\, e^{\left(-\frac{1}{\tau}t\right)}\left[\left(\frac{2L\, i_0}{m\, C\, u_{c0}} + \frac{R}{m}\right)\sin(\omega t) + \cos(\omega t)\right]$$

$$i_c = -i_0 e^{\left(-\frac{1}{\tau}t\right)}\left[\left(\frac{Ri_0 + 2u_{c0}}{mi_0}\right)\sin(\omega t) - \cos(\omega t)\right]$$

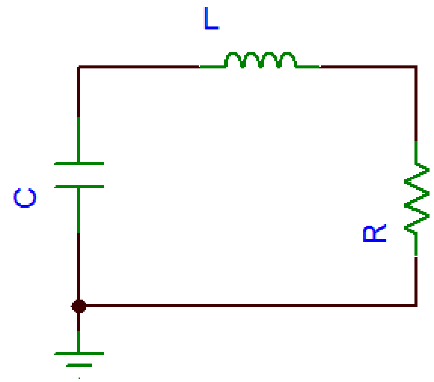
$$\omega = \sqrt{\frac{4L - R^2C}{4CL^2}} = \frac{m}{2L}$$

$$(m = \sqrt{-R^2 + 4L/C})$$

$$\tau = \frac{2L}{R}$$



LCR ZERO-INPUT RESPONSE

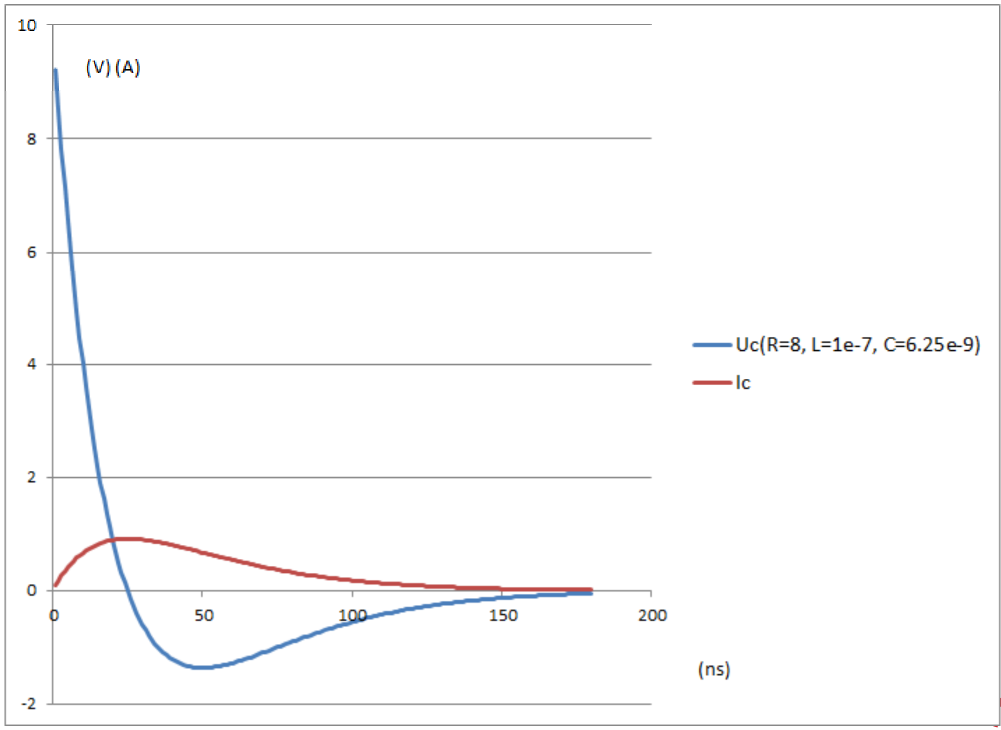


$R^2C - 4L = 0$:

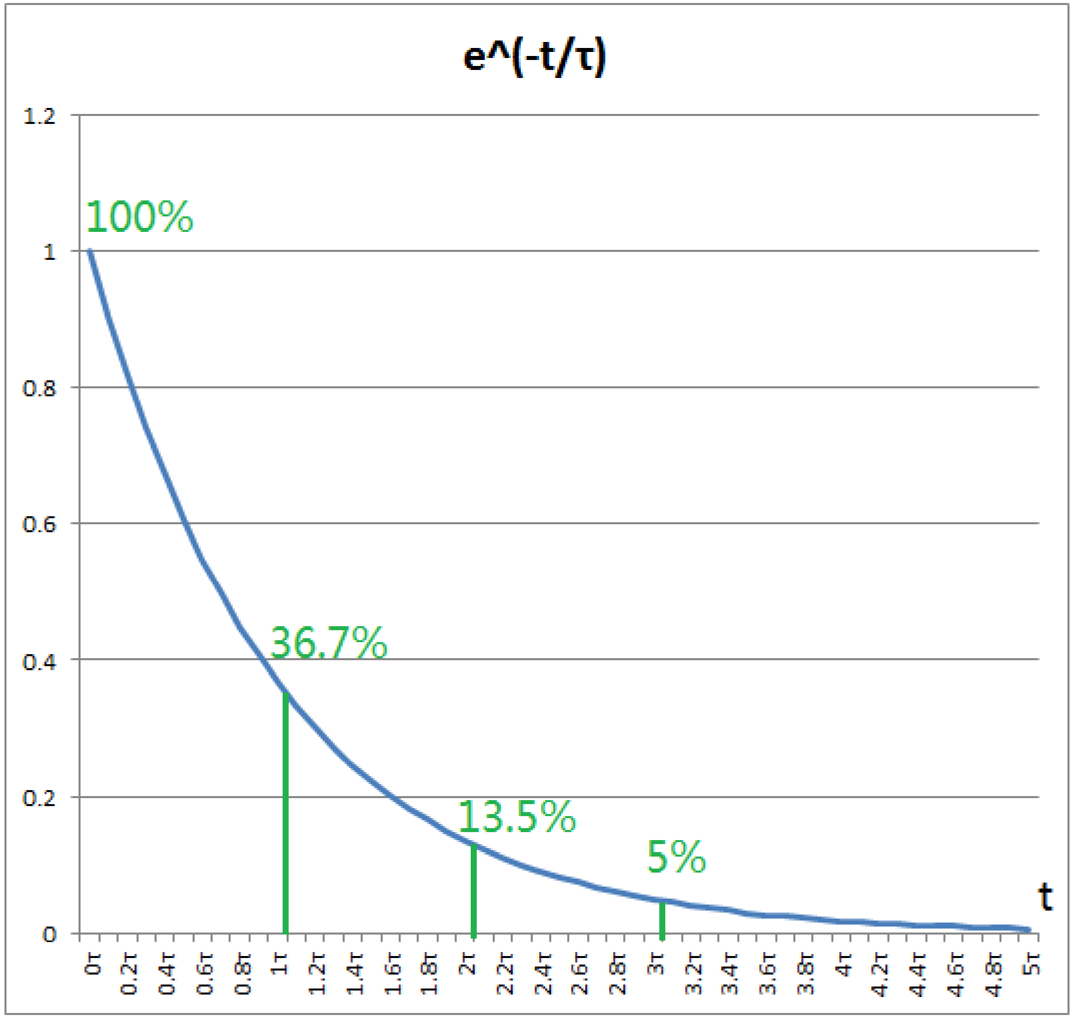
$$u_c = u_{c0}\, e^{\left(-\frac{1}{\tau}t\right)}\left[\left(\frac{i_0}{C\,u_{c0}} + \frac{R}{2\,L}\right)t + 1\right]$$

$$i_c = i_0 e^{\left(-\frac{1}{\tau}t\right)}\left\{\left(-\frac{R}{2L} - \frac{u_{c0}}{Li_0}\right)t + 1\right\}$$

$$\tau = \frac{2\,L}{R} = \frac{RC}{2}$$



TIME CONSTANT "T"



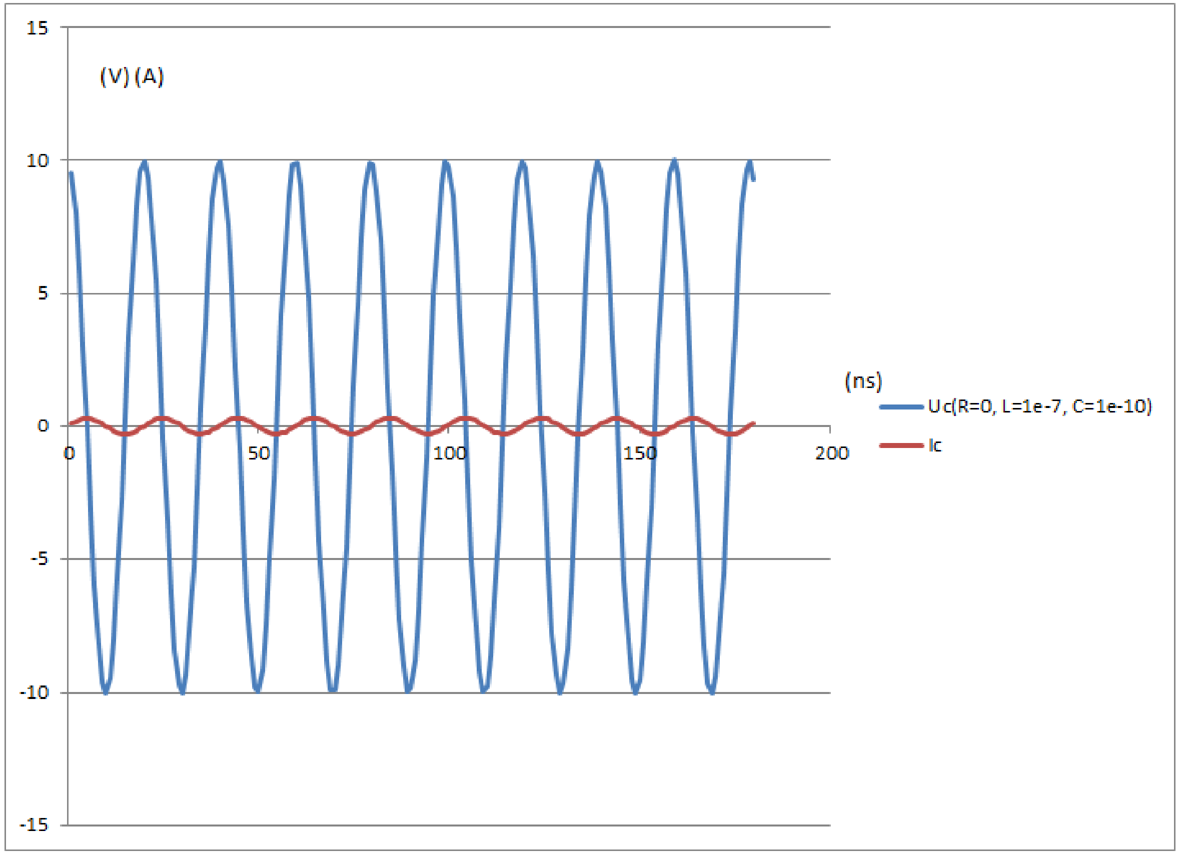
LCR SPECIAL CASE ($R^2 \ll 4\,L/C$)

1) $R^2 \ll \frac{4L}{C}$

$$u_c = u_{c0}\left[\left(\frac{i_0}{\omega\, C\, u_{c0}}\right)\sin(\omega\, t) + \cos(\omega\, t)\right]$$

$$\omega = \sqrt{\frac{4\,L - R^2 C}{4\,C\,L^2}} = \sqrt{\frac{1}{C\,L}}$$

$$\tau = \frac{2\,L}{R} = \infty$$

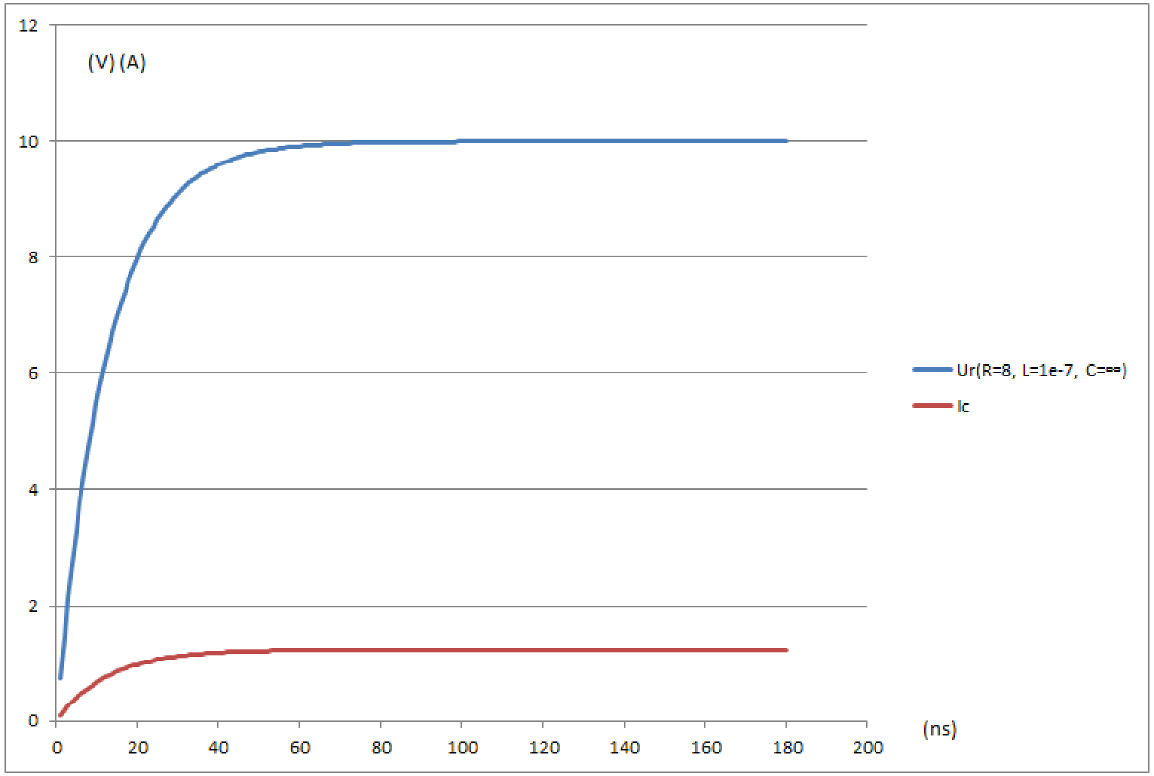


LCR SPECIAL CASE($C \gg R^2/(4\,L)$)

2) $C \gg R^2/(4\,L)$

$$
\begin{aligned}
u_R &= -i_C\,R \\
&= U_0 - (Ri_0 + U_0)e^{-\frac{R}{L}t}
\end{aligned}
$$

$$\tau 1 = \frac{L}{R}$$

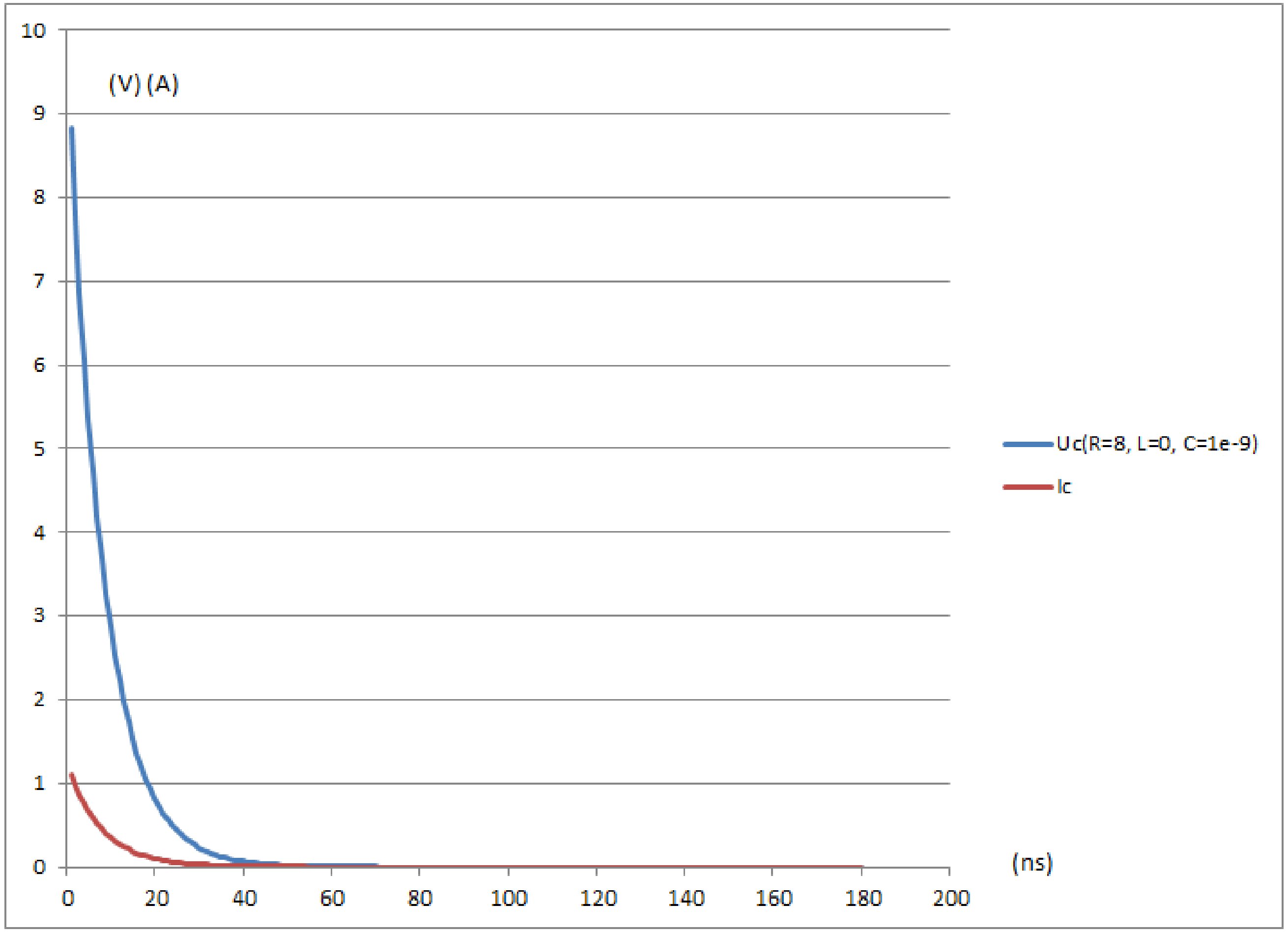


LCR SPECIAL CASE($L \ll R^2C/4$)

3) $L \ll R^2C/4$

$$u_c = u_{c0} e^{\left(-\frac{1}{RC}t\right)}$$

$\tau 1 = RC$



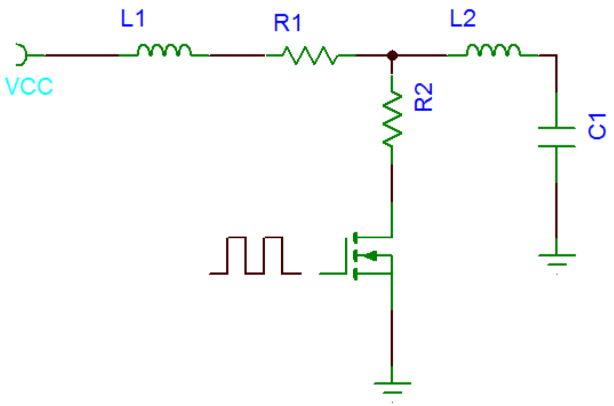
CIRCUIT1

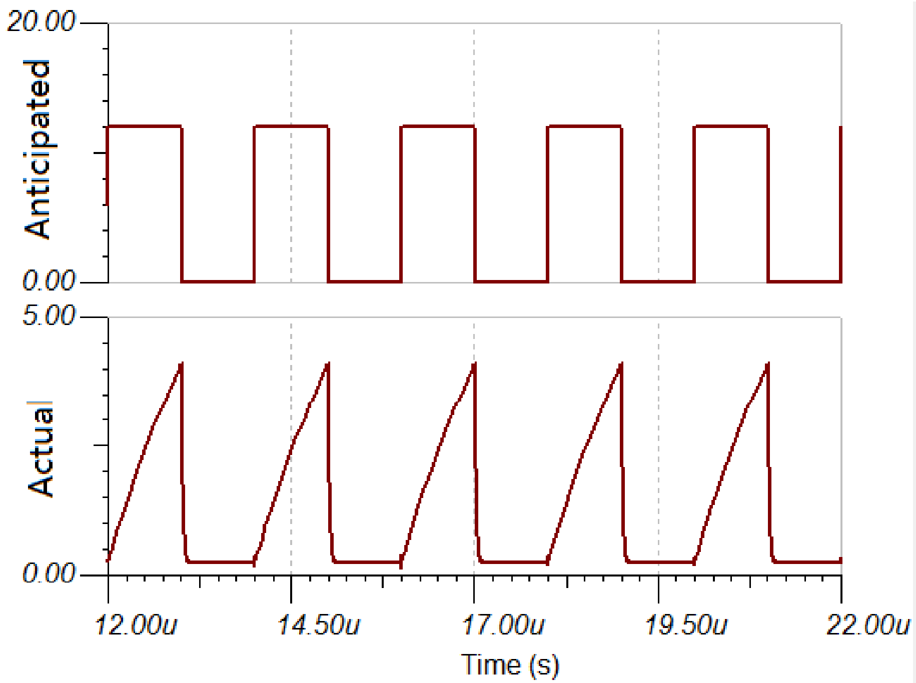
L1=40nH
R1=10Kohm
R2=100ohm
L2=40nH
C1=200pF

1) FET on
$L_2 \ll {R_2}^2 C_1/4$
$\tau\_fal = R_2 C_1 = 20e\text{-}9$

2) FET off
$L_1 + L_2 < {R_1}^2 C_1/4$
$\tau\_ris = R_1 C_1 = 2e\text{-}6$

Solution:
 Smaller R1?
 Narrower trace?
 Buffer?





CIRCUIT2

L1=20nH
R1=1ohm
C1=200pF

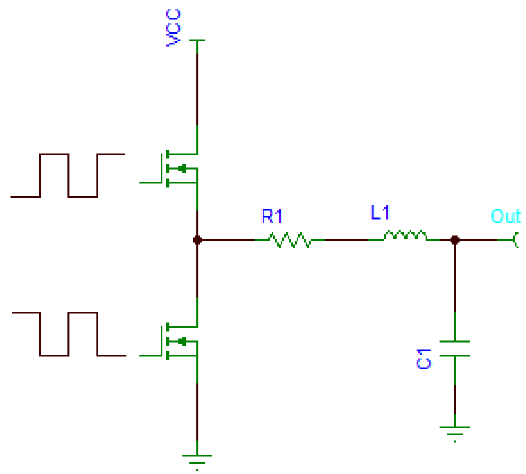
$R_2{}^2 C_1 < 4L_1$

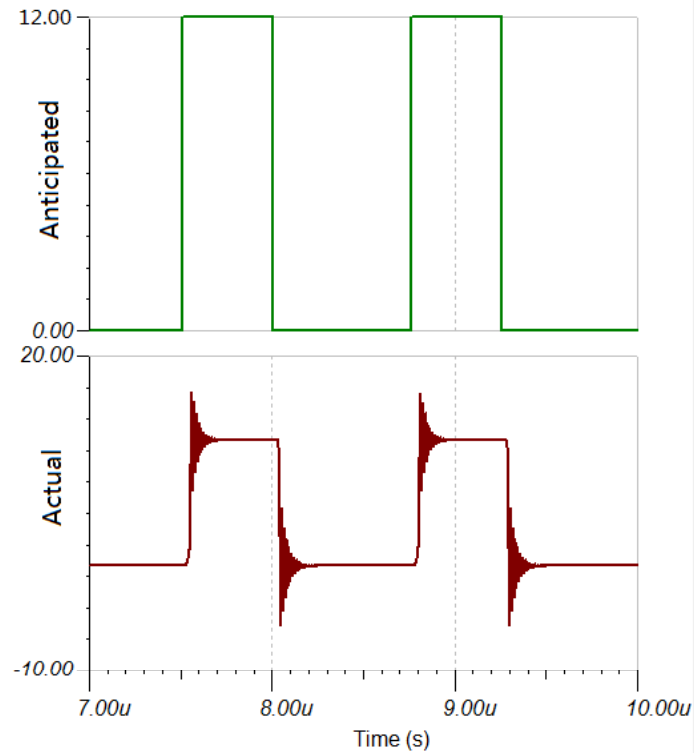
$\tau = \frac{2\,L}{R} = 40ns$

$\omega = \sqrt{\frac{4\,L - R^2 C}{4\,C\,L^2}} = 5e8$

Solution:

Bigger R1?
Narrower trace?
Others?





TRANSMISSION LINE

Assume U_{in} *is sine wave:*

$$U_z = A_1 e^{-jkz} + A_2 e^{jkz}$$
$$I_z = \frac{1}{Z_c}(A_1 e^{-jkz} - A_2 e^{jkz})$$

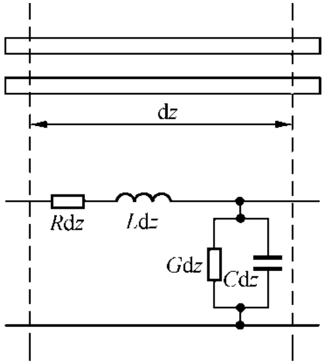
$$Z_C = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

jk= $\sqrt{(R + j\omega L)(G + j\omega C)}$

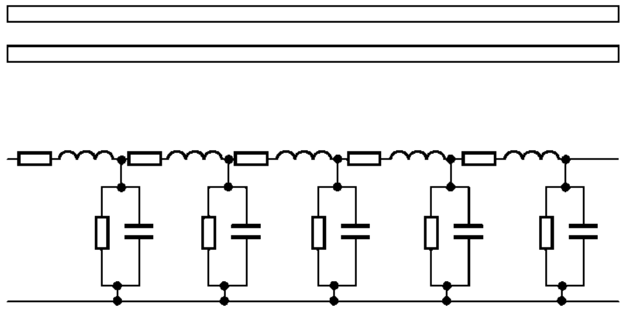
If R<< ωL, G<< ωC :

$Z_C = \sqrt{L/C}$

k = $\beta = \omega \sqrt{LC}$



(a)

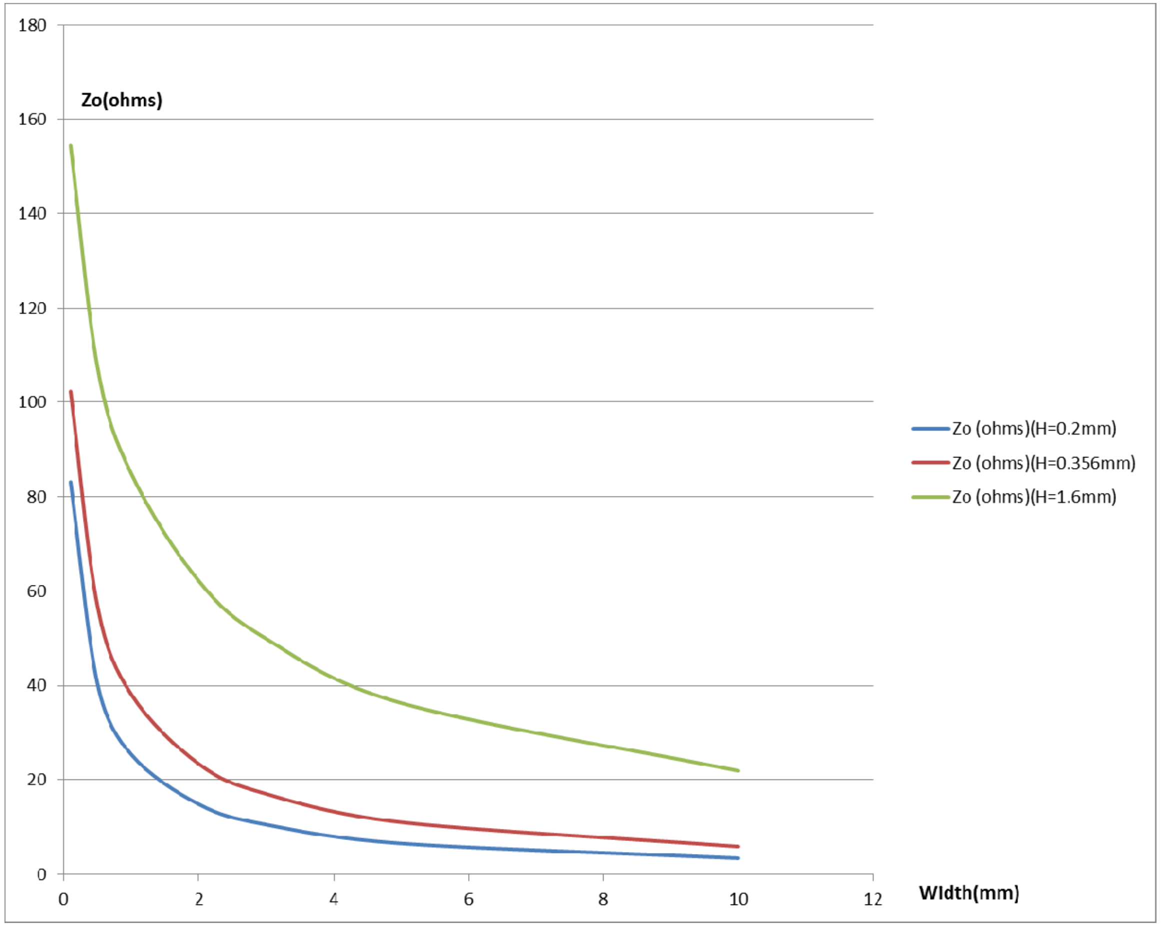


(b)

TRACE CHARACTERISTIC IMPEDANCE

T=35.34um (1OZ)
ER=4.6 (FR-4)

$$Z_C = \sqrt{L/C}$$



IMPEDANCE MATCHING

$$U(l) = \frac{2U_{in}}{\left(\frac{Z_L + Z_C}{Z_L}\right)e^{j\beta l} + \left(\frac{Z_L - Z_C}{Z_L}\right)e^{-j\beta l}} = \frac{U_{in}}{\left(\frac{\cos(\beta l)}{\cos(h)}\right)e^{jh}}$$

$$h = arctg(\frac{z_C tg(\beta l)}{z_L})$$

$$\beta = \omega\ \sqrt{LC}$$

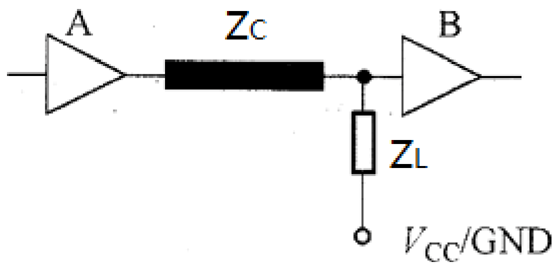
reflection coefficient

$$\Gamma = \frac{z_L - z_C}{z_L + z_C}$$

If $Z_L = Z_C$:

Γ=0 (No reflection)

$$U(l) = U_{in} e^{-j\beta l}$$

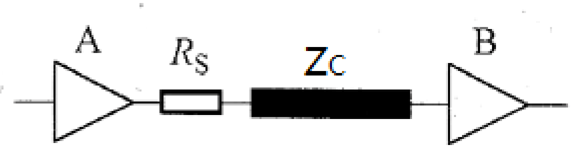


IMPEDANCE MATCHING

$$U(l) = \frac{2U_{in}}{\left(\frac{R_s + Z_C}{Z_C}\right) e^{j\beta l} - \left(\frac{R_s - Z_C}{Z_C}\right) e^{-j\beta l}} = \frac{U_{in}}{\left(\frac{\cos(\beta l)}{\cos(h)}\right) e^{jh}}$$

$h = arctg(\frac{R_s tg(\beta l)}{Z_C})$

$\beta = \omega\sqrt{LC}$



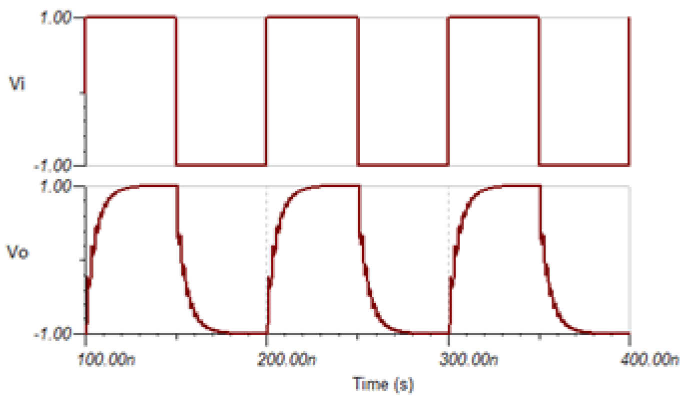
$Z_L = \infty$, $\Gamma = 1$, *total reflection.*

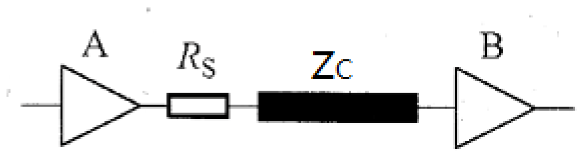
If $R_s = Z_C$:

$U(l) = U_{in} e^{-j\beta l}$

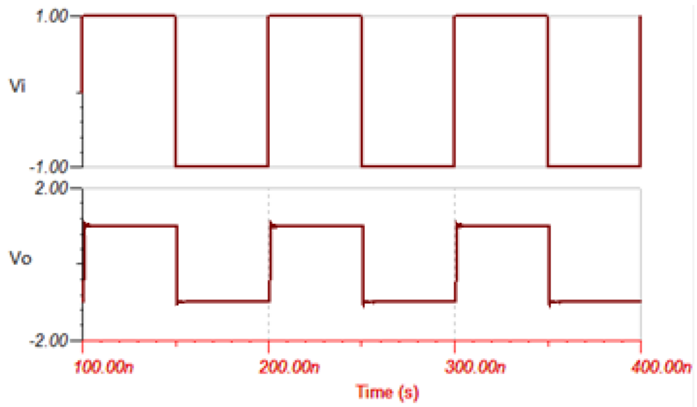
WAVEFORM

$Z_C = 10,\ R_S = 50$

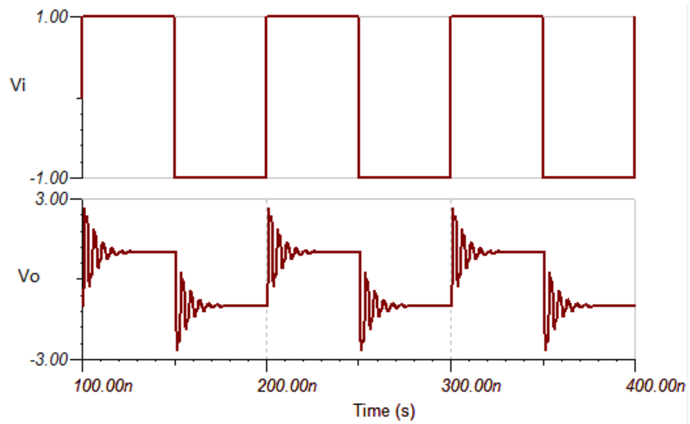




$Z_C = 10,\ R_S = 10$



$Z_C = 10,\ R_S = 2$



PROPAGATION DELAY TIME

propagation speed $v \approx \frac{c}{\sqrt{\varepsilon_r}}$

Propagation delay time Tpd $\approx \frac{1*10^{10}}{v}$ (ps/cm)

FR4, micro-strip, Tpd=56.4ps/cm.

Time shift of two traces is: terr=ΔLength*Tpd

SUMMARY

- Use Saturn to estimate LCR parasitic value.
- Use R, 1/(jωC), jωL to calculate sine signal response.
- For step signal or square signal, use $R^2C - 4L$ to identify properly waveform. Calculate time constant τ .
- If possible, ignore L, C or R to simplify model.
- For high speed or short delay time, use transmission line model. Be aware of impedance matching and length matching.

Thank You